Date:

Chapter: Chapter 4:5 --> Determinants and Cramer's Rule

Objectives: Evaluate determinants

Solve systems of equations by using Cramer's Rule

Notes: $A = \frac{1}{4}s^2\sqrt{3}$

A zoologist tagged a tiger with a GPS tracker so that she could determine the tiger's territory. After several days, the zoologist determined that the tiger's territory was a triangular region. By using the coordinates of the vertices of this triangle, she could use matrices and determinants to determine the size of the tiger's territory. (Do for Ex. 3)

*__Determinant__ = Square array of numbers or variables enclosed between two parallel lines; every square matrix has one.

*__2nd-Order Determinant__ = Determinant of a 2x2 matrix.

How to Find a 2nd-Order Determinant

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

*__3rd-Order Determinant__ = Determinant of a 3x3 matrix.

How to Find a 3rd-Order Determinant

-Use the Diagonal Rule!-

1) Rewrite the 1st 2 columns to the right of the determinant.

2) Draw diagonals of 3 elements beginning with the 1st element. Multiply. Repeat the process beginning with the upper-right element.

3) Find the sum of the products of each diagonal.

4) Subtract the 2nd sum from the 1st.

*__Coefficient Matrix__ (C) = Matrix that contains only coefficients of the system of equations.

Cramer's Rule

-Uses determinants to solve a system of equations-

1) Determinant is a non-zero = solution!

2) Determinant is 0 = no solution or infinity!

3) $\begin{cases} ax + by = m \\ fx + gy = n \end{cases}$ $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$

4) $\begin{cases} ax + by + cz = m \\ fx + gy + hz = n \\ jx + ky + lz = p \end{cases}$ $x = \dfrac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & l \end{vmatrix}}{|C|}$ $y = \dfrac{\begin{vmatrix} a & m & c \\ f & n & h \\ j & p & l \end{vmatrix}}{|C|}$

$$z = \frac{\begin{vmatrix} a & b & m \\ f & g & n \\ j & k & p \end{vmatrix}}{|C|}$$

Area of a Triangle

$$(1/2)\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$ where (a, b), (c, d), and (e, f) are points of a triangle

Examples:

Ex. 1 - Evaluate each determinant.

a) $\begin{vmatrix} -6 & -7 \\ 10 & 8 \end{vmatrix}$

b) $\begin{vmatrix} 7 & 5 \\ 9 & -4 \end{vmatrix}$

Ex. 2 - Evaluate using the Diagonal Rule.

a) $\begin{vmatrix} 3 & -2 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & -3 \end{vmatrix}$

b) $\begin{vmatrix} 3 & -2 & 2 \\ 0 & 4 & 1 \\ -1 & 4 & 5 \end{vmatrix}$

Ex. 3 - Use determinants to find the area of the triangle.

a) From open --> A(0, 0), B(4, 12), C(-2, 8)

b) A(0, 4), B(0, 0), C(6, 0)

Ex. 4 - Solve the system of equations by using Cramer's Rule.

a) $\begin{cases} 5x - 6y = 15 \\ 3x + 4y = -29 \end{cases}$

b) $5x + 4y = 28$

c) $\begin{cases} 2x + 3y = 4 \\ -4x - 6y = 8 \end{cases}$

d) $\begin{cases} 3x - 4y = 2 \\ 6x - 8y = 4 \end{cases}$

e) $\begin{cases} 2x - 3y + 2z = 10 \\ x + 3y + 4z = 14 \\ 3x - y + z = 9 \end{cases}$

f) $x + 2y + z = 4$
$3x + 6y + 3z = 2$
$x - y + z = 3$

Homework:

Average --> p. 226 (#26-36 Evens, 40-46 Evens)

Advanced --> p. 226 (#26-36 Evens, 40-46 Evens, 50, 52, 57, 58, 60, 62-64)