

**Date:**

**Chapter:** Chapter 4:5 --> Determinants and Cramer's Rule

**Objectives:** Evaluate determinants

Solve systems of equations by using Cramer's Rule

**Notes:**

$$A = \frac{1}{4}s^2\sqrt{3}$$

A zoologist tagged a tiger with a GPS tracker so that she could determine the tiger's territory. After several days, the zoologist determined that the tiger's territory was a triangular region. By using the coordinates of the vertices of this triangle, she could use matrices and determinants to determine the size of the tiger's territory. (Do for Ex. 3)

**\*Determinant** = Square array of numbers <sup>or</sup> variables enclosed between two parallel lines; every square matrix has one.

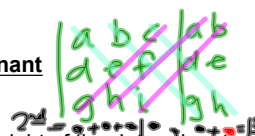
**\*2nd-Order Determinant** = Determinant of a 2x2 matrix.

**How to Find a 2nd-Order Determinant**

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

**\*3rd-Order Determinant** = Determinant of a 3x3 matrix.

**How to Find a 3rd-Order Determinant**



-Use the Diagonal Rule!

- 1) Rewrite the 1st 2 columns to the right of the determinant.
- 2) Draw diagonals of 3 elements beginning with the 1st element. Multiply. Repeat the process beginning with the upper-right element.
- 3) Find the sum of the products of each diagonal.
- 4) Subtract the 2nd sum from the 1st.

**\*Coefficient Matrix (C)** = Matrix that contains only coefficients of the system of equations.

**Cramer's Rule**

-Uses determinants to solve a system of equations-

- 1) Determinant is a non-zero = solution!
- 2) Determinant is 0 = no solution or infinity!

$$3) \begin{cases} ax + by = m \\ fx + gy = n \end{cases} \quad x = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{\begin{vmatrix} a & b \\ f & g \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{\begin{vmatrix} a & b \\ f & g \end{vmatrix}}$$

$$4) \begin{cases} ax + by + cz = m \\ fx + gy + hz = n \\ jx + ky + lz = p \end{cases} \quad x = \frac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ f & g & h \\ j & k & l \end{vmatrix}} \quad y = \frac{\begin{vmatrix} a & m & c \\ f & n & h \\ j & p & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ f & g & h \\ j & k & l \end{vmatrix}}$$

$$z = \frac{\begin{vmatrix} a & b & m \\ f & g & n \\ j & k & p \end{vmatrix}}{\begin{vmatrix} a & b & c \\ f & g & h \\ j & k & l \end{vmatrix}}$$

**Area of a Triangle**

$$(1/2) \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix} \quad \text{where } (a, b), (c, d), \text{ and } (e, f) \text{ are points of a triangle}$$

Examples:

Ex. 1 - Evaluate each determinant.

a)  $\begin{vmatrix} 6 & -7 \\ 10 & 8 \end{vmatrix} = -48 - (-70) = 22$

b)  $\begin{vmatrix} 7 & 5 \\ 9 & -4 \end{vmatrix} = -28 - 45 = -73$

Ex. 2 - Evaluate using the Diagonal Rule.

a)  $\begin{vmatrix} 8 & -2 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix}$   
 $13 - 1 = 12$   
 $5 - 13 = -8$

b)  $\begin{vmatrix} 3 & 2 & 3 \\ 0 & 4 & 4 \\ -4 & 4 & 1 \end{vmatrix}$   
 $20 + 8 + 12 = 40$   
 $-34 - 20 = -54$

Ex. 3 - Use determinants to find the area of the triangle.

a) From open  $\rightarrow A(0, 0), B(4, 12), C(-2, 8)$

$\frac{1}{2} \begin{vmatrix} 0 & 0 & 0 \\ 4 & 12 & 0 \\ -2 & 8 & 0 \end{vmatrix}$   
 $-24 - 0 = -24$   
 $32 - (-24) = 56$   
 $\frac{56}{2} = 28 \text{ m}^2$

b)  $A(0, 4), B(0, 0), C(6, 0)$

$\frac{1}{2} \begin{vmatrix} 0 & 4 & 0 \\ 0 & 0 & 0 \\ 6 & 0 & 0 \end{vmatrix}$   
 $24 - 0 = 24$   
 $\frac{24}{2} = 12 \text{ in}^2$

Ex. 4 - Solve the system of equations by using Cramer's Rule.

a)  $\begin{cases} 5x - 6y = 15 \\ 3x + 4y = -29 \end{cases}$   
 $D = \begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix} = 20 - (-18) = 38$   
 $D_x = \begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix} = 60 - 174 = -114$   
 $D_y = \begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix} = -145 - 45 = -190$   
 $x = \frac{-114}{38} = -3$   
 $y = \frac{-190}{38} = -5$   
 $(-3, -5)$

b)  $5x + 4y = 28$

c)  $\begin{cases} 2x + 3y = 4 \\ -4x - 6y = 8 \end{cases}$   
 $D = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix} = -12 - (-12) = 0$   
 $D_x = \begin{vmatrix} 4 & 3 \\ 8 & -6 \end{vmatrix} = -24 - 24 = -48$   
 $D_y = \begin{vmatrix} 2 & 4 \\ -4 & 8 \end{vmatrix} = 16 - (-16) = 32$   
 $x = \frac{-48}{0} \rightarrow \text{undefined}$

d)  $\begin{cases} 3x - 4y = 2 \\ 6x - 8y = 4 \end{cases}$   
 $D = \begin{vmatrix} 3 & -4 \\ 6 & -8 \end{vmatrix} = -24 - (-24) = 0$   
 $D_x = \begin{vmatrix} 2 & -4 \\ 4 & -8 \end{vmatrix} = -16 - (-32) = 16$   
 $D_y = \begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix} = 12 - 12 = 0$   
 $x = \frac{16}{0} \rightarrow \text{undefined}$

e)  $\begin{cases} 2x - 3y + 2z = 10 \\ x + 3y + 4z = 14 \\ 3x - y + z = 9 \end{cases}$   
 $D = \begin{vmatrix} 2 & -3 & 2 \\ 1 & 3 & 4 \\ 3 & -1 & 1 \end{vmatrix} = 2(-3-4) - 3(1-12) + 2(1-9) = -14 + 33 - 16 = 3$   
 $D_x = \begin{vmatrix} 10 & -3 & 2 \\ 14 & 3 & 4 \\ 9 & -1 & 1 \end{vmatrix} = 10(-3-4) - 3(1-4) + 2(1-27) = -70 + 9 - 52 = -113$   
 $D_y = \begin{vmatrix} 2 & 10 & 2 \\ 1 & 14 & 4 \\ 3 & 9 & 1 \end{vmatrix} = 2(10-36) - 10(1-12) + 2(1-42) = -52 + 110 - 82 = -24$   
 $D_z = \begin{vmatrix} 2 & -3 & 10 \\ 1 & 3 & 14 \\ 3 & -1 & 9 \end{vmatrix} = 2(27-42) - 3(9-42) + 10(9-9) = -30 + 99 + 0 = 69$   
 $x = \frac{-113}{3}$   
 $y = \frac{-24}{3} = -8$   
 $z = \frac{69}{3} = 23$

f)  $\begin{cases} x + 2y + z = 4 \\ 3x + 6y + 3z = 2 \\ x - y + z = 3 \end{cases}$

**Homework:**

Average --> p. 226 (#26-36 Evens, 40-46 Evens)

Advanced --> p. 226 (#26-36 Evens, 40-46 Evens, 50, 52,  
57, 58, 60, 62-64)